Name: ____________________________

Math 10550, Final Exam:
December 15, 2007

Instructor: ________________________

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
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PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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12.	(a)	(b)	(c)	(d)	(e)						
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14.	(a)	(b)	(c)	(d)	(e)						

Name: ______________________________

Instructor: __________________________

Multiple Choice

1.(6 pts.) Find the limit

$$\lim_{x \to 0} \frac{1 - \sqrt{x+1}}{x}.$$

(a) -1

(b) 0

(c) The limit does not exist.

(d) $\frac{1}{3}$

(e) $-\frac{1}{2}$

2.(6 pts.) The function

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

has a removable discontinuity at $x = 2$. We can remove this discontinuity by defining $f(2)$ to be

(a) $\frac{1}{3}$

(b) 1

(c) 0

(d) $\frac{3}{2}$

(e) $\frac{5}{4}$

Name: ______________________________

Instructor: __________________________

3.(6 pts.) If

$$r = \frac{\sin\theta}{1+\cos\theta},$$

then $\dfrac{dr}{d\theta} =$

(a) $\dfrac{\cos\theta + \cos^2\theta - \sin^2\theta}{(1+\cos\theta)^2}$

(b) $\dfrac{1}{1+\cos\theta}$

(c) $-\dfrac{1}{1+\cos\theta}$

(d) $\dfrac{\cos\theta}{(1+\cos\theta)^2}$

(e) $-\dfrac{\cos\theta + \cos^2\theta - \sin^2\theta}{(1+\cos\theta)^2}$

4.(6 pts.) If

$$f(x) = \sqrt{1+\sqrt{1+x}},$$

then $f'(8) =$

(a) $\dfrac{1}{24}$ (b) $\dfrac{1}{12}$ (c) $\dfrac{1}{8}$ (d) $\dfrac{1}{9}$ (e) $\dfrac{1}{2}$

Name: ___________________________

Instructor: ________________________

5.(6 pts.) The **second** derivative of

$$y = (x+1)(x-1)(x^2+1)$$

is

(a) $24x$

(b) $x^2 + 2x - 1$

(c) $12x^2$

(d) $4x^3$

(e) $4x^2 - 2x + 1$

6.(6 pts.) A body travels along a straight line according to the law

$$s = -t^4 - 4t^3 + 20t^2, \quad t \geq 0.$$

At what position, **after** the motion gets started, does the body first come to rest?

(a) $s = 32$ (b) $s = 36$ (c) $s = 2$

(d) $s = 12$ (e) $s = 24$

Name: ______________________________

Instructor: __________________________

7.(6 pts.) The equation of the tangent line to the curve

$$y = x^3 + 6x^2 + 10x + 6$$

at $x = -2$ is

(a) $y = \frac{x}{2}$

(b) $y = -2x - 2$

(c) $y = -\frac{1}{2}x + 1$

(d) $y = -x + 2$

(e) $y = -2x$

8.(6 pts.) Use the implicit differentiation to find the equation of the tangent line to the curve

$$\sqrt{5x + 9y} = 2 + xy^2 + y$$

at the point $(0, 1)$.

(a) $y = \frac{4}{3}x + 1$

(b) $y = -\frac{5}{6}x$

(c) $y = \frac{1}{3}x + 1$

(d) $y = -\frac{5}{6}x + 1$

(e) $y = \frac{1}{3}x$

Name: ______________________________

Instructor: __________________________

9.(6 pts.) A cylinder is carved out of ice and then left in the sun to melt. If the radius decreases at a rate of 3 inches per hour and the height decreases at a rate of 6 inches per hour, how fast is the surface area of the cylinder decreasing when the cylinder is at height 5 feet and radius one foot? (Hint: 12 inches in a foot.)

Answer: The **total** surface area decreases at a rate of

(a) $\frac{3\pi}{4}$ ft^2/hr (b) $\frac{5\pi}{4}$ ft^2/hr (c) $\frac{5\pi}{2}$ ft^2/hr

(d) $\frac{9\pi}{2}$ ft^2/hr (e) 2π ft^2/hr

10.(6 pts.) Use linear approximation to estimate

$$\frac{1}{\sqrt{4.1}}.$$

(a) $\frac{1}{\sqrt{4.1}} \approx \frac{81}{160}$ (b) $\frac{1}{\sqrt{4.1}} \approx \frac{39}{80}$ (c) $\frac{1}{\sqrt{4.1}} \approx \frac{9}{20}$

(d) $\frac{1}{\sqrt{4.1}} \approx \frac{79}{160}$ (e) $\frac{1}{\sqrt{4.1}} \approx \frac{41}{80}$

Name: ______________________________

Instructor: __________________________

11.(6 pts.) The maximum and minimum values of

$$f(x) = \frac{x}{x^2+1},$$

on the interval [0,2] are

(a) $M = \frac{1}{2},\ m = 0$

(b) $M = \frac{1}{2},\ m = -\frac{1}{2}$

(c) $M = 1,\ m = -\frac{3}{25}$

(d) $M = \frac{2}{5},\ m = 0$

(e) $m = 0$ is a minimum; there is no maximum.

12.(6 pts.) Determine the number of solutions of the equation

$$x^3 - 15x + 1 = 0$$

in the interval $[-2, 2]$. The number of solutions is

(a) 2 (b) 0 (c) 1 (d) 3 (e) 4

Name: ______________________________

Instructor: __________________________

13.(6 pts.) Consider the function

$$f(x) = \frac{x^2+3}{x-1}.$$

One of the following statements is true. Which one?

(a) The line $y = x+1$ is a slant asymptote of f, and the line $x = 1$ is a vertical asymptote of f.

(b) f has no horizontal or slant asymptotes, and the line $x = -1$ is a vertical asymptote.

(c) The line $y = 0$ is a horizontal asymptote of f, and the line $x = -1$ is a vertical asymptote of f.

(d) The line $y = x+2$ is a slant asymptote of f, and the line f has no vertical asymptotes.

(e) The line $y = x-1$ is a slant asymptote of f and the line $x = 1$ is a vertical asymptote of f.

14.(6 pts.) Consider the function

$$f(x) = \frac{x^2+3}{x-1}.$$

One of the following statements is true. Which one?

(a) f is increasing on the interval $(-1, 3)$.

(b) f has a local minimum at $x = -1$.

(c) f is decreasing on the intervals $(-1, 1)$ and $(1, 3)$.

(d) f is increasing on the intervals $(-\infty, -1)$ and $(1, 3)$.

(e) f has a local minimum at $x = 1$.

Name: ______________________________

Instructor: __________________________

15.(6 pts.) Consider the function

$$f(x) = \frac{\sqrt{9x^6 - x}}{x^3 + 1}.$$

One of the following statements is true. Which one?

(a) $y = 3$ is a horizontal asymptote of f, and $y = -3$ is not a horizontal asymptote.

(b) f has no horizontal asymptotes.

(c) $y = 0$ and $y = -3$ are both horizontal asymptotes of f.

(d) $y = \pm 3$ are both horizontal asymptotes of f.

(e) $y = 0$ is a horizontal asymptote of f.

16.(6 pts.) The function $f(x) = (2x + 1)^4 - 24x^2 + 5x$ is concave down on which of the following intervals?

(a) $(0, 1)$

(b) $(-1, \infty)$

(c) $(-\infty, -1)$

(d) $(-1, 0)$

(e) $(-\infty, 1)$

Name: ____________________________

Instructor: __________________________

17.(6 pts.) An open box is to be made from a square of side one by cutting four identical squares near the vertices. The box with the largest **volume** has a **height** of

(a) $\frac{1}{6}$ (b) $\frac{3}{4}$ (c) $\frac{2}{17}$

(d) $\frac{1}{2}$ (e) $\frac{1}{4}$

18.(6 pts.) When applying Newton's method to approximate a root of the equation $x^3 - x + 2 = 0$, with initial guess $x_1 = 1$, the value of x_2 is:

(a) 1.5 (b) 0.5 (c) 0

(d) 2 (e) 3

Name: ____________________________

Instructor: ________________________

19.(6 pts.) Which of the following is a Riemann sum corresponding to the integral

$$\int_2^3 x^4 dx \ ?$$

(a) $\dfrac{2}{n}\sum_{i=1}^{n}(2+\dfrac{i}{n})^4$ (b) $\dfrac{1}{n}\sum_{i=1}^{n}(2+\dfrac{i}{n})^4$ (c) $\dfrac{1}{2n}\sum_{i=1}^{n}(\dfrac{i}{n})^4$

(d) $\dfrac{2}{n}\sum_{i=1}^{n}(\dfrac{2+i}{n})^4$ (e) $\dfrac{1}{n}\sum_{i=1}^{n}(\dfrac{i}{n})^4$

20.(6 pts.) A function $f(x)$ defined on the interval $[-1,1]$ has an antiderivative $F(x)$. Assume that $F(-1) = 8$ and $F(1) = 7$. Which one of the statements below is true?

(a) $\int_{-1}^{1} f(x)dx = 1.$

(b) $F(x)$ is an increasing function.

(c) $f(x)$ can be an odd function.

(d) $\int_{-1}^{1} f(x)dx = 0.$

(e) $\int_{-1}^{1} f(x)dx = -1.$

Name: ____________________________

Instructor: ________________________

21.(6 pts.) Calculate the integral

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |\sin x| dx.$$

(a) π (b) 1 (c) $\frac{\pi}{2}$

(d) 2π (e) 2

22.(6 pts.) The volume of the solid obtained by rotating the region given by $x^2 + y^2 = 1$, $x \geq 0$ and $y \geq 0$, about the line $y = -1$ is

(a) $\pi \int_0^1 (1 - x^2) dx$

(b) $\pi \int_0^1 [1 - x^2 + 2\sqrt{1 - x^2}] dx$

(c) $2\pi \int_0^1 x[1 - x^2 + 2\sqrt{1 - x^2}] dx$

(d) $2\pi \int_0^1 x\sqrt{1 - x^2} dx$

(e) $\pi \int_0^1 (1 + \sqrt{1 - x^2})^2 dx$

Name: ____________________________

Instructor: _________________________

23.(6 pts.) Find the volume of the solid obtained by rotating about the y-axis the region between $y = x^2$ and $y = x^4$.

(a) $\frac{\pi}{6}$ (b) π (c) $\frac{\pi}{10}$ (d) 2π (e) $\frac{\pi}{5}$

24.(6 pts.) Find the average of $f(x) = \sin^2(x) \cdot \cos(x)$ over $[0, \frac{\pi}{2}]$.

(a) $\frac{2}{\pi}$ (b) $\frac{1}{3\pi}$ (c) $\frac{2}{3\pi}$

(d) $\frac{1}{3}$ (e) $\frac{1}{\pi}$

Name: ____________________________

Instructor: ________________________

25.(6 pts.) A (vertical) cylindrical tank has a height 1 meter and base radius 1 meter. It is filled full with a liquid with a density 100 kg/m^3. Find the work required to empty the tank by pumping all of the liquid to the top of the tank.

(a) 500π kg-m

(b) 100π kg-m

(c) 200π kg-m

(d) 0 kg-m

(e) 50π kg-m

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